



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DIVISION ERRORS OF THE REPSOLD MERIDIAN
CIRCLE.

BY R. H. TUCKER.

The determination of the errors of graduation of a divided circle offers one of the most refined problems in practical astronomy.

Notwithstanding the excellence of modern workmanship in the construction of such circles, it may be assumed that there will always remain errors, systematic or accidental, or both, in the position of the divisions.

Whether these are large enough to be a matter of concern depends not only upon the care and exactness with which the circle has been divided, but upon the accuracy aimed at in the determinations for which it is to be used.

For field instruments, it may in general be taken for granted that the errors left by any first-class maker are within the limits required in the use of the circles.

The best meridian circles now made, with the advantages which the constructors have gained from their own experience, and from the tests and investigation of circles actually in use, have probably very small outstanding errors. Still, it is important to ascertain, for any particular instrument, what degree of fineness has been reached, and to show what the probable inaccuracy of a determination, for instance, of a star's position will be, due to this source of error. The definite determination of every division of the circle would then give the means of freeing observation nearly completely from the effect of this error; not entirely, for the determination itself will necessarily have its residual of error, which should, however, be small in comparison with errors to be measured.

In observing practice it has sometimes been the custom to use a circle movable upon its axis, and to shift from time to time, so that for the various observations of any star different sets of divisions will have been employed, and the mean result will be affected by the mean of the errors of such divisions.

By observing with the circle fixed in respect to the telescope, but reversing the whole instrument, we obtain two distinct sets for the determinations of a star; or four, if each observation is

made upon the pair of divisions nearest the reference point in each reading microscope.

While reversal of the instrument is intended for control and elimination of other sources of error, it has the effect of reducing in the mean result the error due to inaccurate divisions. By comparison of results of observation, knowing the probable error of circle reading, bisection, and from other sources, it is possible to arrive at an approximate estimate of the effect of division error. But it will be far more satisfactory in any discussion of work of high standard to be able to account for every source of error by independent investigation, and to show that the resulting probable error can be traced definitely to all sources.

The investigation only of a sufficiently large number of graduations to show the effect of division error upon the observation of star places will at least give definite data; while, as the process must consist, in the main, of a series of steps, each depending upon those preceding, whatever is done affords a basis for continuation. The present plan for the REPSOLD meridian circle includes the investigation of every degree of the fixed circle, which is the one ordinarily employed in observation. The problem in full is to determine the exact angle between any two divisions marking the degrees. For this purpose, any particular division may be adopted as the reference mark, and the angles of the others are then to be measured from it. Two divisions, 180° apart, may be taken as a standard diameter, and the angles to other diameters may be measured; or the mean of two diameters at right angles giving a set of four divisions ninety degrees apart, may be adopted, and all other sets of four be referred to them.

If it is the custom to read the circle by all four of the microscopes, the last solution will give the data needed. This is the plan of the present undertaking; but the material thus accumulated affords all that is necessary for the determination of each separate division of the four. The readings would need to be corrected for eccentricity and flexure, and the positions and changes in the individual microscopes followed up.

There are two methods for the determination of division error, either of which may be employed with this instrument. One may use simultaneous readings of the two circles, for investigations of the errors of one or both.

Or, by a pair of extra microscopes, which may be set at any

angle desired with respect to the four ordinarily used, the errors of one circle may be investigated.

The first process has been the one employed thus far; while, in further work, the other may be adopted. Before outlining the process of observing, some account of the instrument should be given. The circles are two feet in diameter, each being divided to every 2', there being about 150 divisions to the inch. This affords an idea of the scale; an arc of 1'' upon the circle is represented in lineal measure by $\frac{1}{18000}$ inch.

The circles are read by microscopes of twenty-six inches focal length, having an effective magnifying power of thirty-five diameters. Each microscope screw carries a pair of threads about 25'' apart, the screw-heads being divided into sixty parts, affording easy estimates of tenths, or very closely 0''.1.

The circles are graduated in the same direction, increasing opposite to the hands of a watch as one faces the circle. One of them is movable upon its axis, permitting of easy and quick adjustment in any position with respect to the other, or fixed, circle.

The general outline of observing is then to apply successively the arcs of one circle, as a measure to similar arcs of the other. By the application, for instance, of all arcs of 3° in length of one circle, to all arcs of 3° upon the other, the errors in the length of each arc of both circles is determined. And for either circle the errors of its 3° arcs are entirely independent of the errors of the other circle.

This can perhaps be made plain by an illustration.

(a) If we had an exact yard-stick, and wished to measure an exact foot, if we divided the yard into three approximately equal parts, and applied each of them in succession, the mean of the three measures would be an exact foot. The parts, of course, except for convenience, we will say, in making the measurement, need not even be approximately of the same length; their sum will be a yard, and the mean one foot.

(b) The resulting exact foot could then be used to mark off three equal divisions upon the yard-stick.

The problem then resolves itself into that of making our comparisons as accurately as possible. Our exact standard is the entire circumference of either circle, by which the aliquot parts of the other are to be measured.

It will be clear that no assumption is made of errors balancing

in their effect; nor that the average of any number of division errors, however large, will be zero.

The process, on the contrary, gives, within the error of observation, the difference in angle between any two of the degree marks, upon the circle fully measured.

By reading the two circles simultaneously in one position, and then moving the telescope, carrying them both through any angle to another position, the difference of the two measures of the angle will be due to a combination of the following causes:

First.—Personal error in each reading. As far as personal equation is concerned, this should have no effect, since, with constant illumination and no variable conditions, it should affect all readings alike.

There remains the accidental inaccuracy, or probable error, to which every observation of every character is subject, to a varying degree. It can never be eliminated; but an increasing accuracy can be attained up to a certain practicable limit, by increase of the number of readings.

Second.—Changes in the relation of the two circles to each other, when the position of each is changed. This may be due to the flexures of the circles, and these should consequently be determined and corrected for, if necessary; or, better, the readings can be so arranged that the effect will be eliminated. By flexure here referred to, that of a twisting character is meant. The flexure, resulting from a compression or flattening of the circles, will always be eliminated from the mean of four microscopes; just as the eccentricity of each circle, which will affect every microscope reading, in the two positions, will be eliminated in the mean of four.

The twisting effect may be due to some especial weakness of the rim, or to an unequal distribution of weight. It can be assumed to be of the same character as would be produced by attaching a small weight to the rim of a circle, which would evidently vary in its effect depending upon the position of the circle. Tests were made in the investigation of division error at the DUDLEY Observatory, to show this practically.

The circle flexure, once determined in amount, can be corrected for by means of two terms, acting respectively in proportion to the sine and cosine of the angle between any position of the circle and some adopted position. Thus, taking the position of the fixed circle when the telescope points to the zenith, the

flexure correction can be determined and applied for any other position, in terms of sine and cosine $Z. D.$ The flexures of both circles have been accordingly determined.

The effect is eliminated from the mean of two readings, in positions of the circle 180 degrees apart; and, in order to reduce all systematic corrections, with their corresponding errors, the system of reading in the two opposite positions has been carried out in the present work.

Third.—Changes may also occur in the shape of one of the circles, thus altering its relation to the microscopes; or one or more of the microscopes may change in position with respect to the others. Such changes are observed as between successive nights, and even during the course of a night's observing, and are traceable, somewhat obscurely, to changes of temperature in general.

They can be guarded against by making readings in a series, forward and back, so that the detection of change is possible by comparison. Any progressive change is also eliminated in the mean of the two readings, since the means of all made in the same positions fall at nearly the same epoch.

The series should, however, be short, and anything that would tend to produce a change should be avoided. The excellent ventilation of the meridian-circle room, and the isolation of the heat of the lamps, leave nothing to be desired in this respect. The change in temperature in the room has been almost invariably the slight fall due at the time of day when readings are generally made.

No series of readings has ever more than barely exceeded one hour, and careful test of all the series has never shown any one which gave evidence of weakness in this respect. Even if a decided progressive movement were shown, it would be safer, in practice, to observe the series again. For the object in view involves the discussion of quantities extremely small, even in comparison with the generally precise results arrived at in work with this instrument, and it is only by attention to every detail of observation and reduction that accumulation of small errors is to be avoided.

There remains, *fourth*, the graduation errors of all the divisions upon which readings were made in the two positions. Except for accidental personal error, all other sources of error can be determined, corrected for, if necessary, or mostly eliminated by proper arrangement of the observations.

For the determination of the graduation errors of the fixed circle, the movable circle is read simultaneously through a series of positions. Then the movable circle is shifted on its axis, and again a series is made. The errors of both circles are obtained by the proper combination of these readings.

Thus, for the 45° points, each circle was read at 0° and 45° , in four positions. Then the movable circle was shifted 45° ; and this was repeated, in all, eight times. This gave the division error of 45° on each circle, or, rather, the difference between 0° and 45° , which will be referred to simply as the 45° division error.

The actual error of 0° is not needed; it has been assumed ".00 for convenience. Any other value could have been assumed. It is sometimes taken large enough to produce resulting errors for the other divisions, which shall all be of the same sign.

The 45° divisions were determined in both positions of the instrument, fixed circle west, and east, for additional weight. For as error accumulates in the successive steps, it is important to have the first ones strong.

Next follows the series, in which the movable circle is shifted 15° each time; and the series consists of readings every 15° through an arc of 90° . The opposite quadrant has to be taken in a separate series, in order that none shall exceed proper length.

The next step is down to 3° . A satisfactory expedient for this step has been to shift the circle 9° at a time, and read every 9° through an arc of 90° . Three series are observed in each position, beginning, respectively, at 0° , 3° , and 6° . This gives the same strength to the determinations, by the comparison of each 3° with two 15° divisions already measured, as if the series extended only through 30° and was observed at every 3° . And for either circle, the division errors are independent of those adopted for the other circle.

It will be noted that the 15° series gives also a new determination of the 45° arcs. Similarly, the 3° series gives further determinations of the 15° arcs. But, in the reduction, these later measures have been considered as checks only, and the measures, as made, have been carried forward without change, for which there seems to be no evident need.

Thus far the form has been that given in the illustration (a) of the standard yard; and every 3° of both circles has been measured.

For the determination of the single or 1° divisions of the fixed circle, the illustration (*b*) seems more apt. One of the 3° arcs of B, the movable circle, will be used throughout; each of the four 1° divisions will be compared with all of the ninety divisions of A, the fixed circle. This will give a determination of these four divisions of B, which will be actually independent of the errors of A; and these four will probably be the best-known divisions upon either circle. Since their measured errors will now be applied, in obtaining the errors of A, the fixed-circle divisions will no longer be entirely independent of the errors of B. There will be, for instance, five separate determinations of each unknown 1° division of A; three will depend upon measured 3° divisions of A and B; the remaining two will also depend upon the two newly measured 1° divisions of B.

The reduction of the observations is, in general, of a simple character. But the greatest care is necessary to avoid errors, which would be quite insignificant were it not that they might accumulate. Every individual reading of a microscope has had its test scrutiny, both to show that there could be no mistake in the original observation, and to check, by its relation to the mean, the accuracy of the mean itself. Every mean of microscopes has been taken by two separate methods; and the differences of each from the mean of four has been compared for identical or opposite positions of the circle, to control any effect of change.

Whenever any list of figures has been transferred a new mean is taken, simply as a check upon the copy itself.

As regards the observing, each series stands alone, about one hour in length. The repetition of readings forward and back has always been adhered to. The settings have been made within $10''$, without any stress being laid upon closer adjustment, to avoid the effect of Runs correction, and of irregularities of screws. Each determination rests upon series made in opposite quadrants, for the elimination of circle flexure from the mean.

The results of investigation of the 3° arcs of both circles, already complete, show an average division error for the mean of four microscopes of $0''.17$, with no single set much exceeding $0''.5$.

From this it can be predicted that the circles have probably an average division error for single divisions of about $0''.4$, and that there will be no single divisions with errors much larger than $1''$. Referring back to the size of the circles, the inference is

that the largest error of graduation of a single division will be about $\frac{1}{18000}$ inch; while the average will be about $\frac{1}{50000}$ inch.

Nearly 4000 complete readings of each circle are to be made to carry out the investigation as now planned and well advanced towards completion.

It is anticipated that the probable error of a final determination of division error, increasing, as it necessarily does, with each step in the subdivision, will not much exceed $\pm''.05$ for the set of four 1° marks.

This would give for the probable error of the determination of single divisions about $\pm 0''.1$, corresponding to $\frac{1}{180000}$ inch on the scale of the circle.

These figures indicate the refinement of the workmanship upon the instrument. That of the determinations may be illustrated in another way. The scratches upon the silver rim, delicate as they appear to the eye, are yet about $10''$ in width, as as seen in the microscopes.

In the progress of this work, it is planned to measure the position of the center of that delicate line, within a probable error of the one-hundredth part of its width; and this is to be done with respect to other lines which cannot be seen in the same field of view, but must be reached by movement of the whole instrument.

For the prosecution of this work by the simultaneous reading of both circles, I have had the volunteer assistance of Professor R. G. AITKEN, which has been given in time which was his own. The exactness of the results depends upon both observers equally, and it gives me the greatest pleasure to acknowledge his efficiency and his good will.

There is possibly no class of observing more monotonous or demanding more rigorous and persistent care in small details. And the results are so long in forthcoming, also, that one lacks the spur of accomplishment by the way; for it is not until any set of subdivisions has been completely carried out that any results are definitely obtained.

The corrections for circle flexure are represented by the expressions:

$$\begin{aligned}\text{Circle A : } &+''.08 \sin (R - 315^\circ) +''.04 \cos (R - 315^\circ); \\ \text{Circle B : } &+''.02 \sin (R - 315^\circ) +''.05 \cos (R - 315^\circ);\end{aligned}$$

when R is the reading of either circle, at the lower left hand

microscope. The fixed circle A reads 315° when the telescope points to the zenith.

These flexures were determined with each circle in both positions, east and west. The corrections will need to be considered in the reduction of the readings for the horizontal flexure of the telescope, and can be combined with that correction into one term depending upon the zenith distance.

The eccentricity resulting from a discussion of the 45° arcs is $2''.61$ for circle A.

The average eccentricity for B is $1''.81$; but this will need to be combined with the eccentricity $0''.9$ of the movable circle with respect to the collar in which it turns, for any particular position.